Platonic Solids

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1 Discussion

This brief note describes the 5 Platonic solids and lists specific vertex values and face connectivity indices that allow you to build triangle or polygon meshes of the solids. In each of the sections the following notation is used.

v	number of vertices	A	dihedral angle between adjacent faces
e	number of edges	R	radius of circumscribed sphere
f	number of faces	r	radius of inscribed sphere
p	number of edges per face	L	edge length
q	number of edges sharing a vertex	S	surface area
		V	volume

The following are identities for the polyhedra.

$$\sin(A/2) = \frac{\cos\left(\frac{\pi}{q}\right)}{\sin\left(\frac{\pi}{p}\right)}, \quad \frac{S}{L^2} = \frac{fp\cot\left(\frac{\pi}{p}\right)}{4}, \quad V = \frac{rS}{3},$$

$$\frac{R}{L} = \frac{\tan\left(\frac{\pi}{q}\right)\tan\left(\frac{A}{2}\right)}{2}, \quad \frac{r}{L} = \frac{\cot\left(\frac{\pi}{p}\right)\tan\left(\frac{A}{2}\right)}{2}, \quad \frac{R}{r} = \tan\left(\frac{\pi}{p}\right)\tan\left(\frac{\pi}{q}\right)$$

Tetrahedron

Parameters:

$$
\begin{aligned}
v &= 4 & \sin(A) &= \sqrt{8}/3 \\
e &= 6 & \cos(A) &= 1/3 \\
f &= 4 & R/L &= \sqrt{6}/4 \\
p &= 3 & r/L &= \sqrt{6}/12 \\
q &= 3 & S/L^2 &= \sqrt{3} \\
 & & V/L^3 &= \sqrt{2}/12
\end{aligned}
$$

Unit length vertices:

$$
\begin{aligned}
v_0 &= (0, 0, 1) & v_2 &= (-\sqrt{2}/3, \sqrt{6}/3, -1/3) \\
v_1 &= (2\sqrt{2}/3, 0, -1/3) & v_3 &= (-\sqrt{2}/3, -\sqrt{6}/3, -1/3)
\end{aligned}
$$

Triangle connectivity:

$$
(0, 1, 2) \qquad (0, 2, 3) \qquad (0, 3, 1) \qquad (1, 3, 2)
$$

Hexahedron (cube)

Parameters:

$v = 8$	$\sin(A) = 1$
$e = 12$	$\cos(A) = 0$
$f = 6$	$R/L = \sqrt{3}/2$
$p = 4$	$r/L = 1/2$
$q = 3$	$S/L^2 = 6$
	$V/L^3 = 1$

Unit length vertices:

$v_0 = (-1, -1, -1)/\sqrt{3}$	$v_4 = (-1, -1, 1)/\sqrt{3}$
$v_1 = (1, -1, -1)/\sqrt{3}$	$v_5 = (1, -1, 1)/\sqrt{3}$
$v_2 = (1, 1, -1)/\sqrt{3}$	$v_6 = (1, 1, 1)/\sqrt{3}$
$v_3 = (-1, 1, -1)/\sqrt{3}$	$v_7 = (-1, 1, 1)/\sqrt{3}$

Triangle connectivity:

$(0, 3, 2)$	$(0, 2, 1)$	$(0, 1, 5)$	$(0, 5, 4)$
$(0, 4, 7)$	$(0, 7, 3)$	$(6, 5, 1)$	$(6, 1, 2)$
$(6, 2, 3)$	$(6, 3, 7)$	$(6, 7, 4)$	$(6, 4, 5)$

Face connectivity (faces are squares):

$(0, 3, 2, 1)$	$(0, 1, 5, 4)$	$(0, 4, 7, 3)$
$(6, 5, 1, 2)$	$(6, 2, 3, 7)$	$(6, 7, 4, 5)$

Octahedron

Parameters:

$$
\begin{aligned}
&v = 6 && \sin(A) = \sqrt{8}/3 \\
&e = 12 && \cos(A) = -1/3 \\
&f = 8 && R/L = \sqrt{2}/2 \\
&p = 3 && r/L = \sqrt{6}/6 \\
&q = 4 && S/L^2 = 2\sqrt{3} \\
& && V/L^3 = \sqrt{2}/3
\end{aligned}
$$

Unit length vertices:

$$
\begin{aligned}
&v_0 = (1, 0, 0) && v_3 = (0, -1, 0) \\
&v_1 = (-1, 0, 0) && v_4 = (0, 0, 1) \\
&v_2 = (0, 1, 0) && v_5 = (0, 0, -1)
\end{aligned}
$$

Triangle connectivity:

$$
\begin{array}{cccc}
(4, 0, 2) & (4, 2, 1) & (4, 1, 3) & (4, 3, 0) \\
(5, 2, 0) & (5, 1, 2) & (5, 3, 1) & (5, 0, 3)
\end{array}
$$

Dodecahedron

Parameters:

$v = 20$	$\sin(A) = 2/\sqrt{5}$
$e = 30$	$\cos(A) = -1/\sqrt{5}$
$f = 12$	$R/L = \sqrt{3}(\sqrt{5}+1)/4$
$p = 5$	$r/L = \sqrt{250+110\sqrt{5}}/20$
$q = 3$	$S/L^2 = 3\sqrt{25+10\sqrt{5}}$
	$V/L^3 = (15+7\sqrt{5})/4$

Unit length vertices, $a = 1/\sqrt{3}$, $b = \sqrt{(3-\sqrt{5})/6}$, $c = \sqrt{(3+\sqrt{5})/6}$:

$v_0 = (a, a, a)$	$v_5 = (-a, a, -a)$	$v_{10} = (b, -c, 0)$	$v_{15} = (-c, 0, -b)$
$v_1 = (a, a, -a)$	$v_6 = (-a, -a, a)$	$v_{11} = (-b, -c, 0)$	$v_{16} = (0, b, c)$
$v_2 = (a, -a, a)$	$v_7 = (-a, -a, -a)$	$v_{12} = (c, 0, b)$	$v_{17} = (0, -b, c)$
$v_3 = (a, -a, -a)$	$v_8 = (b, c, 0)$	$v_{13} = (c, 0, -b)$	$v_{18} = (0, b, -c)$
$v_4 = (-a, a, a)$	$v_9 = (-b, c, 0)$	$v_{14} = (-c, 0, b)$	$v_{19} = (0, -b, -c)$

Triangle connectivity:

$(0, 8, 9)$	$(0, 9, 4)$	$(0, 4, 16)$	$(0, 12, 13)$	$(0, 13, 1)$	$(0, 1, 8)$
$(0, 16, 17)$	$(0, 17, 2)$	$(0, 2, 12)$	$(8, 1, 18)$	$(8, 18, 5)$	$(8, 5, 9)$
$(12, 2, 10)$	$(12, 10, 3)$	$(12, 3, 13)$	$(16, 4, 14)$	$(16, 14, 6)$	$(16, 6, 17)$
$(9, 5, 15)$	$(9, 15, 14)$	$(9, 14, 4)$	$(6, 11, 10)$	$(6, 10, 2)$	$(6, 2, 17)$
$(3, 19, 18)$	$(3, 18, 1)$	$(3, 1, 13)$	$(7, 15, 5)$	$(7, 5, 18)$	$(7, 18, 19)$
$(7, 11, 6)$	$(7, 6, 14)$	$(7, 14, 15)$	$(7, 19, 3)$	$(7, 3, 10)$	$(7, 10, 11)$

Face connectivity (faces are pentagons):

$(0, 8, 9, 4, 16)$	$(0, 12, 13, 1, 8)$	$(0, 16, 17, 2, 12)$	$(8, 1, 18, 5, 9)$
$(12, 2, 10, 3, 13)$	$(16, 4, 14, 6, 17)$	$(9, 5, 15, 14, 4)$	$(6, 11, 10, 2, 17)$
$(3, 19, 18, 1, 13)$	$(7, 15, 5, 18, 19)$	$(7, 11, 6, 14, 15)$	$(7, 19, 3, 10, 11)$

Icosahedron

Parameters:

$v = 12$	$\sin(A) = 2/3$
$e = 30$	$\cos(A) = -\sqrt{5}/3$
$f = 20$	$R/L = \sqrt{10 + 2\sqrt{5}}/4$
$p = 3$	$r/L = \sqrt{42 + 18\sqrt{5}}/12$
$q = 5$	$S/L^2 = 5\sqrt{3}$
	$V/L^3 = 5(3 + \sqrt{5})/12$

Unit length vertices, $t = (1 + \sqrt{5})/2$, $s = \sqrt{1 + t^2}$:

$v_0 = (t, 1, 0)/s$	$v_3 = (-t, -1, 0)/s$	$v_6 = (-1, 0, t)/s$	$v_9 = (0, -t, 1)/s$
$v_1 = (-t, 1, 0)/s$	$v_4 = (1, 0, t)/s$	$v_7 = (-1, 0, -t)/s$	$v_{10} = (0, t, -1)/s$
$v_2 = (t, -1, 0)/s$	$v_5 = (1, 0, -t)/s$	$v_8 = (0, t, 1)/s$	$v_{11} = (0, -t, -1)/s$

Triangle connectivity:

$(0, 8, 4)$	$(0, 5, 10)$	$(2, 4, 9)$	$(2, 11, 5)$	$(1, 6, 8)$
$(1, 10, 7)$	$(3, 9, 6)$	$(3, 7, 11)$	$(0, 10, 8)$	$(1, 8, 10)$
$(2, 9, 11)$	$(3, 11, 9)$	$(4, 2, 0)$	$(5, 0, 2)$	$(6, 1, 3)$
$(7, 3, 1)$	$(8, 6, 4)$	$(9, 4, 6)$	$(10, 5, 7)$	$(11, 7, 5)$