Intersection of a Sphere and a Cone

David Eberly, Geometric Tools, Redmond WA 98052
https://www.geometrictools.com/

Created: March 8, 2002
Last Modified: September 11, 2020

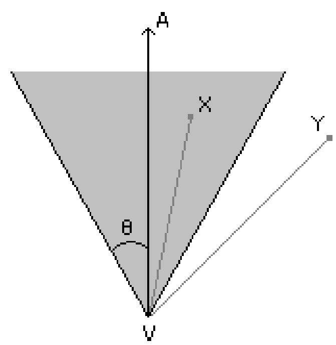
Contents

1 Definitions

A sphere with center $\mathbf{C}$ and radius $r > 0$ is defined by the set of points $\mathbf{X}$ satisfying $|\mathbf{X} - \mathbf{C}| = r$. The solid sphere is the sphere plus the region it bounds, specified as $|\mathbf{X} - \mathbf{C}| \leq r$.

An *infinite single-sided cone* with vertex $\mathbf{V}$, axis ray with origin at $\mathbf{V}$ and unit-length direction $\mathbf{A}$, and cone angle $\theta \in (0, \pi/2)$ is defined by the set of points $\mathbf{X}$ such that the vector $\mathbf{X} - \mathbf{V}$ forms an angle θ with $\mathbf{A}$. The algebraic condition is $\mathbf{A} \cdot (\mathbf{X} - \mathbf{V}) = |\mathbf{X} - \mathbf{V}| \cos\theta$. The *infinite solid cone* is the cone plus the region it bounds, specified as $\mathbf{A} \cdot (\mathbf{X} - \mathbf{V}) \geq |\mathbf{X} - \mathbf{V}| \cos\theta$. Figure 1 shows an infinite solid cone.

Figure 1. A 2D view of a single-sided, infinite, solid code. The boundary of the cone is drawn with black lines, but the top boundary does not have a black line, which indicates the cone extends to infinity. The point $\mathbf{X}$ is inside the solid cone and the point $\mathbf{Y}$ is outside the solid cone.



Because of the constraint on θ, both $\cos\theta > 0$ and $\sin\theta > 0$.

A parametric representation of the infinite cone is

$$\mathbf{X}(h, \phi) = \mathbf{V} + h\mathbf{A} + (h \tan\theta)(\cos\phi \mathbf{W}_0 + \sin\phi \mathbf{W}_1) \tag{1}$$

where $\{\mathbf{W}_0, \mathbf{W}_1, \mathbf{A}\}$ is a right-handed orthonormal set; that is, the vectors of the set are unit length, mutually perpendicular and $\mathbf{A} = \mathbf{W}_0 \times \mathbf{W}_1$. The parameters are constrained by $h \in [0, +\infty)$ and $\phi \in [0, 2\pi)$. The variable h is referred to as *height* of the cone. A parametric representation of the infinite solid cone is

$$\mathbf{X}(h, \phi, \rho) = \mathbf{V} + h\mathbf{A} + \rho(\cos\phi \mathbf{W}_0 + \sin\phi \mathbf{W}_1) \tag{2}$$

where the h and ϕ constraints are the same as for the infinite cone and where $\rho \in [0, h \tan\theta]$.

The infinite cone can be truncated with one or two planes perpendicular to the cone axis. To distinguish between them, I name the objects as shown in Table 1.

Table 1. Various types of cones.

cone name	height constraints
infinite cone	$h \in [0, +\infty)$
infinite truncated cone	$h \in [h_{\min}, +\infty)$ for finite $h_{\min} > 0$
finite cone	$h \in [0, h_{\max}]$ for finite $h_{\max} > 0$
cone frustum	$h \in [h_{\min}, h_{\max}]$ for finite $h_{\min}$ and $h_{\max}$ with $0 < h_{\min} < h_{\max}$

The remainder of the document describes the test-intersection queries between a sphere and the four types of cones. The problem of determining whether a sphere intersects a cone is equivalent to using Minkowski sums, where the sphere is shrunk to its center point $\mathbf{C}$ by its radius r and the cone is expanded to a *sphere-swept volume*. This volume is formed by placing a spheres of radius r with centers at the points of the cone, an infinite union of volumes so to speak. The test-intersection query becomes a point-in-sphere-swept-volume test.

2 Intersection of a Sphere with an Infinite Cone

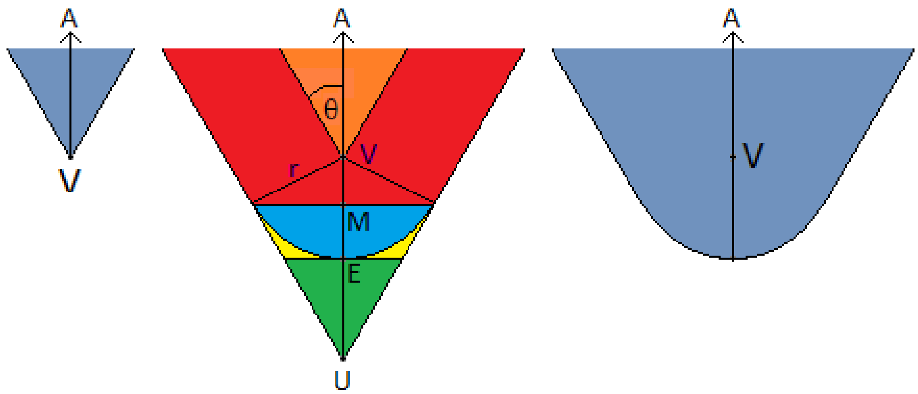
The sphere-swept volume for the infinite cone lives in a *supercone* defined by

$$\mathbf{A} \cdot (\mathbf{X} - \mathbf{U}) \geq |\mathbf{X} - \mathbf{U}| \cos\theta \tag{3}$$

where $\mathbf{U} = \mathbf{V} - (r/\sin\theta)\mathbf{A}$. If the sphere center is outside the supercone, then the sphere and infinite solid cone do not intersect. When the center is inside the supercone, additional tests must be applied to determine intersection. To avoid the square root calculation on the right-hand side of equation (3), the test is equivalent to the two tests $\mathbf{A} \cdot (\mathbf{X} - \mathbf{U}) \geq 0$ and $(\mathbf{A} \cdot (\mathbf{X} - \mathbf{U}))^2 \geq |\mathbf{X} - \mathbf{U}|^2 \cos^2\theta$.

Figure 2 shows various regions of interest in a cross section of the cone. The cross section lives in a plane containing the sphere center $\mathbf{C}$, the cone vertex $\mathbf{V}$ and the cone axis direction $\mathbf{A}$.

Figure 2. The absence of a black boundary line at the top of each of the objects indicates that there is no boundary and the object interiors extend to infinity. Left: The infinite truncated solid cone. Right: The sphere-swept infinite truncated solid cone. Middle: A partitioning of the sphere-swept infinite truncated solid cone. The sphere-swept volume is formed by the orange, red, blue and violet regions. The yellow and green regions are outside the sphere-swept volume. The supercone is the union of all the colored regions.



The points are $\mathbf{M} = \mathbf{V} - (r\sin\theta)\mathbf{A}$ and $\mathbf{E} = \mathbf{V} - r\mathbf{A}$. The plane through $\mathbf{M}$ and perpendicular to the cone axis contains the circle of tangency between the supercone and the vertex sphere.

The conditions for when the sphere will or will not intersect the infinite solid cone are as follows. The tests are executed in the order specified.

1. $\mathbf{C}$ is outside the supercone. The center is in the white region of Figure 2. The outside test is $\mathbf{A}\cdot(\mathbf{C}-\mathbf{U}) \geq |\mathbf{C}-\mathbf{U}|\cos\theta$.
2. $\mathbf{C}$ is outside the sphere-swept infinite cone when it is below the plane through $\mathbf{E}$ and perpendicular to the cone axis. The center is in the green region of Figure 2. The outside test is $\mathbf{A}\cdot(\mathbf{C}-\mathbf{E}) < 0$, which is equivalent to $\mathbf{A}\cdot(\mathbf{C}-\mathbf{V}) < -r$.
3. $\mathbf{C}$ is inside the sphere-swept volume when it is above or on the plane through $\mathbf{M}$ and perpendicular to the cone axis. The center is in the red or orange regions of Figure 2. The inside test is $\mathbf{A}\cdot(\mathbf{C}-\mathbf{M}) \geq 0$, which is equivalent to $\mathbf{A}\cdot(\mathbf{C}-\mathbf{V}) \geq -r\sin\theta$.
4. At this time we know that $\mathbf{C}$ is in the blue or yellow regions of Figure 2. The inside test is $|\mathbf{C}-\mathbf{M}|^2 \leq r^2$.

Listing 1 contains pseudocode for the test-intersection query using the alternate algorithm.

Listing 1. Pseudocode for the test-intersection query between a sphere and an infinite cone using the alternate algorithm. The function returns `true` if and only if there is an intersection.

```
bool SphereIntersectsInfiniteCone(Sphere sphere, Cone cone)
{
    Vector3 U = cone.vertex - (sphere.radius * cone.sinReciprocal) * cone.axis;
    Vector3 CmU = sphere.center - U;
    Real AdCmU = Dot(cone.axis, CmU);
    if (AdCmU > 0)
```

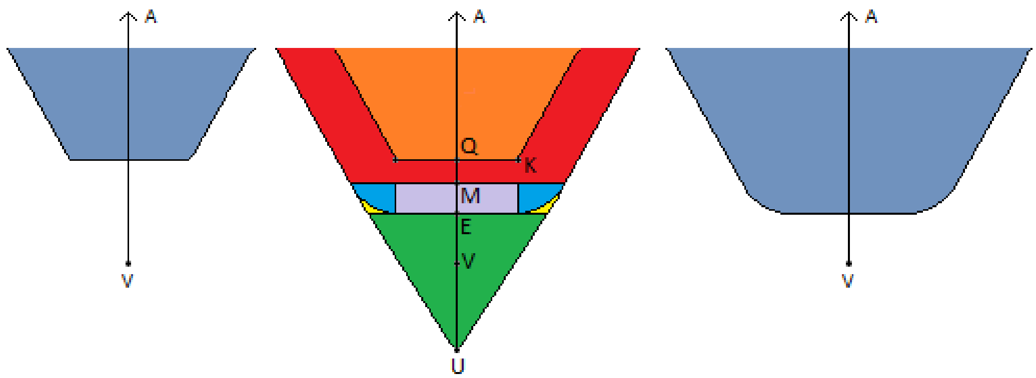
```
    {
        Real sqrLengthCmU = Dot(CmU, CmU);
        if (AdCmU * AdCmU >= sqrLengthCmU * cone.cosAngleSqr)
        {
            // The center is inside the supercone.
            Vector3 CmV = sphere.center - cone.vertex;
            Real AdCmV = Dot(cone.axis, CmV);
            if (AdCmV < -sphere.radius)
            {
                // The center is outside the sphere-swept volume (green region)
                return false;
            }

            Real rSinAngle = sphere.radius * cone.sinAngle;
            if (AdCmV >= -rSinAngle)
            {
                // The center is inside the sphere-swept volume (red or orange region).
                return true;
            }

            Real sqrLengthCmV = Dot(CmV, CmV);
            if (sqrLengthCmV <= sphere.radiusSqr)
            {
                // The center is inside the sphere-swept volume (blue region).
                return true;
            }
            else
            {
                // The center is outside the sphere-swept volume (yellow region).
                return false;
            }
        }
    }

    // The center is outside the supercone (white region).
    return false;
}
```

---

3 Intersection of a Sphere with an Infinite Truncated Cone

Figure 3 shows regions of interest in a cross section of the cone. The cross section lives in a plane containing the sphere center **C**, the cone vertex **V** and the cone axis direction **A**.

Figure 3. The absence of a black boundary line at the top of each of the objects indicates that there is no boundary and the object interiors extend to infinity. Left: The infinite truncated solid cone. Right: The sphere-swept infinite truncated solid cone. Middle: A partitioning of the sphere-swept infinite truncated solid cone. The sphere-swept volume is formed by the orange, red, blue and violet regions. The yellow and green regions are outside the sphere-swept volume.



The points are $\mathbf{Q} = \mathbf{V} + h_{\min}\mathbf{A}$, $\mathbf{E} = \mathbf{Q} - r\mathbf{A}$ and $\mathbf{M} = \mathbf{Q} - (r \sin\theta)\mathbf{A}$. The point $\mathbf{K}$ is the center of one of the spheres placed along the circle of cone points on the truncation plane at $h_{\min}$. The union of these spheres has a circle of points tangent to the sphere-swept volume, where the black line separating the red and blue regions intersects that volume. The blue regions are defined by the spheres at the $\mathbf{K}$ points.

A query for the sphere center $\mathbf{C}$ when located in the blue or yellow regions will process the point $\mathbf{K}$ closest to $\mathbf{C}$. Define $\mathbf{\Delta} = \mathbf{C} - \mathbf{Q}$. The unit-length vector from $\mathbf{Q}$ in the direction of $\mathbf{K}$ is the normalized projection of $\mathbf{\Delta}$ onto a plane perpendicular to the cone axis, call this direction

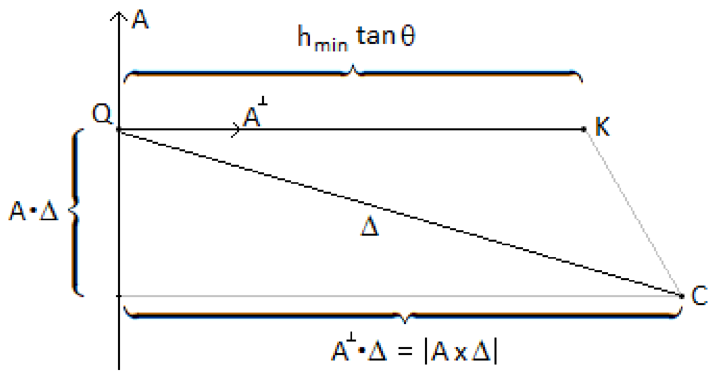
$$\mathbf{A}^{\perp} = \frac{\mathbf{\Delta} - (\mathbf{A} \cdot \mathbf{\Delta})\mathbf{A}}{|\mathbf{\Delta} - (\mathbf{A} \cdot \mathbf{\Delta})\mathbf{A}|} = \frac{\mathbf{\Delta} - (\mathbf{A} \cdot \mathbf{\Delta})\mathbf{A}}{|\mathbf{A} \times \mathbf{\Delta}|} \tag{4}$$

It follows that

$$\mathbf{K} = \mathbf{Q} + (h_{\min} \tan\theta)\mathbf{A}^{\perp}, \quad \mathbf{C} = \mathbf{Q} + (\mathbf{A} \cdot \mathbf{\Delta})\mathbf{A} + |\mathbf{A} \times \mathbf{\Delta}|\mathbf{A}^{\perp} \tag{5}$$

Figure 4 shows the geometric configuration described here.

Figure 4. The geometric configuration showing sphere center $\mathbf{C}$ and the closest point $\mathbf{K}$ on the tangent circle.



The conditions for when the sphere will or will not intersect the infinite truncated solid cone are as follows. The tests are executed in the order specified.

1. $\mathbf{C}$ is outside the supercone. The center is in the white region of Figure 3. The outside test is $\mathbf{A} \cdot (\mathbf{C} - \mathbf{U}) \geq |\mathbf{C} - \mathbf{U}| \cos\theta$.

2. $\mathbf{C}$ is outside the sphere-swept volume when it is below the plane through $\mathbf{E}$ and perpendicular to the cone axis. The center is in the green region of Figure 3. The outside test is $\mathbf{A} \cdot (\mathbf{C} - \mathbf{E}) < 0$, which is equivalent to $\mathbf{A} \cdot (\mathbf{C} - \mathbf{V}) < -r$.

3. $\mathbf{C}$ is inside the sphere-swept volume when it is above or on the plane through $\mathbf{M}$ and perpendicular to the cone axis. The center is in the red or orange regions of Figure 3. The inside test is $\mathbf{A} \cdot (\mathbf{C} - \mathbf{M}) \geq 0$, which is equivalent to $\mathbf{A} \cdot (\mathbf{C} - \mathbf{V}) \geq -r \sin\theta$.

4. $\mathbf{C}$ is inside the sphere-swept volume if it is above the $\mathbf{E}$-plane, below the $\mathbf{M}$-plane and a distance d from the cone axis with $d = |\mathbf{A} \times \mathbf{\Delta}| \leq h_{\min} \tan\theta$. The center is in the violet region of Figure 3. Because we have already executed the tests in items (1), (2) and (3), the inside test requires only the distance comparison.

5. The final test is whether $\mathbf{C}$ is in a yellow or a blue region of Figure 3. This requires a simple distance comparison between the sphere radius and the distance from $\mathbf{C}$ to $\mathbf{K}$. The test

$$|\mathbf{C} - \mathbf{K}|^2 = (\mathbf{A} \cdot \mathbf{\Delta})^2 + (|\mathbf{A} \times \mathbf{\Delta}| - h_{\min} \tan\theta)^2 \leq r^2$$

is true if and only if $\mathbf{C}$ is inside the sphere centered at $\mathbf{K}$, in the blue region. When the test is false, $\mathbf{C}$ is in the yellow region and outside the sphere-swept volume.

Listing 2 contains pseudocode for the test-intersection query.

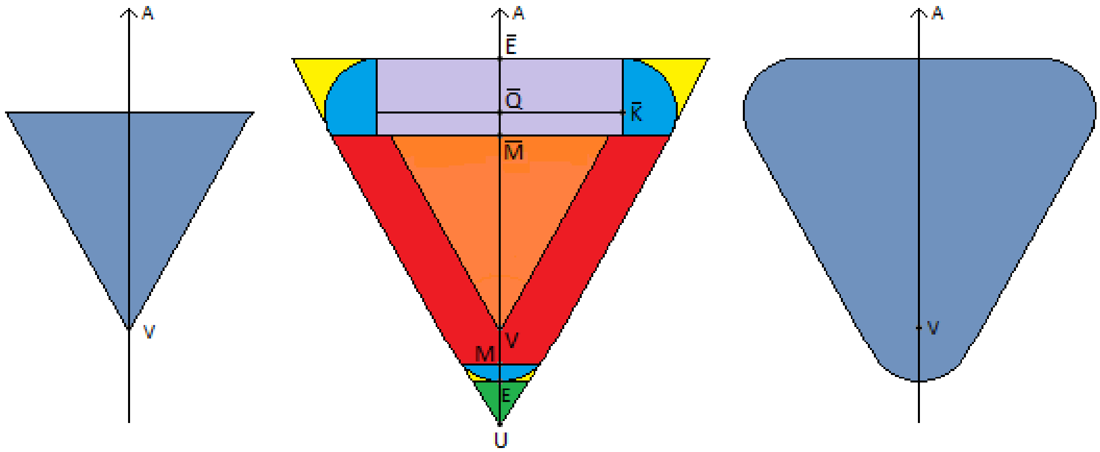
Listing 2. Pseudocode for the test-intersection query between a sphere and an infinite truncated cone. The function returns `true` if and only if there is an intersection.

```
bool SphereIntersectsInfiniteTruncatedCone(Sphere sphere, Cone cone)
{
    Vector3 U = cone.vertex - (sphere.radius * cone.sinReciprocal) * cone.axis;
    Vector3 CmU = sphere.center - U;
    Real AdCmU = Dot(cone.axis, CmU);
    if (AdCmU > 0)
    {
        Real sqrLengthCmU = Dot(CmU, CmU);
        if (AdCmU * AdCmU >= sqrLengthCmU * cone.cosAngleSqr)
        {
            // The center is inside the supercone.
            Vector3 CmV = sphere.center - cone.vertex;
            Real AdCmV = Dot(cone.axis, CmV);
            if (AdCmV < cone.minHeight - sphere.radius)
            {
                // The center is outside the sphere-swept volume (green region).
                return false;
            }

            Real rSinAngle = sphere.radius * cone.sinAngle;
            if (AdCmV >= -rSinAngle)
            {
                // The center is inside the sphere-swept volume (red or orange region).
                return true;
            }

            Vector3 D = CmV - cone.minHeight * cone.axis;  // = C - Q = C - V - hmin * A
            Real lengthAxD = Length(Cross(cone.axis, D));
            Real hminTanAngle = cone.minHeight * cone.tanAngle;
            if (lengthAxD <= hminTanAngle)
            {
                // The center is inside the sphere-swept volume (violet region).
                return true;
            }

            Real AdD = AdCmV - cone.minHeight;  // = Dot(A, C - Q) = Dot(A, C - V) - hmin
            Real diff = lengthAxD - hminTanAngle;
            Real sqrLengthCmK = AdD * AdD + diff * diff;
            if (sqrLengthCmK <= sphere.radiusSqr)
            {
                // The center is inside the sphere-swept volume (blue region).
                return true;
            }
            else
            {
                // The center is outside the sphere-swept volume (yellow region).
                return false;
            }
        }
    }

    // The center is outside the supercone (white region).
    return false;
}
```

4 Intersection of a Sphere with a Finite Cone

Figure 5 shows regions of interest in a cross section of the cone. The cross section lives in a plane containing the sphere center **C**, the cone vertex **V** and the cone axis direction **A**.

Figure 5. Left: The finite solid cone. Right: The sphere-swept finite solid cone. Middle: A partitioning of the sphere-swept finite solid cone. The sphere-swept volume is formed by the orange, red, blue and violet regions. The yellow and green regions are outside the sphere-swept volume.



The points are $\mathbf{E} = \mathbf{V} - r\mathbf{A}$, $\mathbf{M} = \mathbf{V} - (r\sin\theta)\mathbf{A}$, $\overline{\mathbf{Q}} = \mathbf{V} + h_{\max}\mathbf{A}$, $\overline{\mathbf{E}} = \overline{\mathbf{Q}} + r\mathbf{A}$ and $\overline{\mathbf{M}} = \overline{\mathbf{Q}} - (r\sin\theta)\mathbf{A}$. The point $\overline{\mathbf{K}}$ is the center of one of the spheres placed along the circle of cone points on the truncated plane at $h_{\max}$. The union of these spheres has a circle of points tangent to the sphere-swept volume, where the black line separating the red and blue regions intersects that volume. The blue regions are defined by the spheres at the $\overline{\mathbf{K}}$ points.

A query for the sphere center $\mathbf{C}$ when located in the blue or yellow regions above the $h_{\max}$ plane will process the point $\mathbf{K}$ closest to $\mathbf{C}$. Define $\overline{\boldsymbol{\Delta}} = \mathbf{C} - \overline{\mathbf{Q}}$. The unit-length vector from $\overline{\mathbf{Q}}$ in the direction of $\overline{\mathbf{K}}$ is the normalized projection of $overline\boldsymbol{\Delta}$ onto a plane perpendicular to the cone axis, call this direction

$$\mathbf{A}^{\perp} = \frac{\overline{\boldsymbol{\Delta}} - (\mathbf{A}\cdot\overline{\boldsymbol{\Delta}})\mathbf{A}}{|\overline{\boldsymbol{\Delta}} - (\mathbf{A}\cdot\overline{\boldsymbol{\Delta}})\mathbf{A}|} = \frac{\overline{\boldsymbol{\Delta}} - (\mathbf{A}\cdot\overline{\boldsymbol{\Delta}})\mathbf{A}}{|\mathbf{A}\times\overline{\boldsymbol{\Delta}}|} \tag{6}$$

It follows that

$$\overline{\mathbf{K}} = \overline{\mathbf{Q}} + (h_{\max}\tan\theta)\mathbf{A}^{\perp}, \quad \mathbf{C} = \overline{\mathbf{Q}} + (\mathbf{A}\cdot\overline{\boldsymbol{\Delta}})\mathbf{A} + |\mathbf{A}\times\overline{\boldsymbol{\Delta}}|\mathbf{A}^{\perp} \tag{7}$$

The geometric configuration is similar to the one shown in Figure 4. The conditions mentioned next also use $\boldsymbol{\Delta} = \mathbf{C} - \mathbf{Q}$.

The conditions for when the sphere will or will not intersect the finite solid cone are as follows. The tests are executed in the order specified.

1. $\mathbf{C}$ is outside the supercone. The center is in the white region of Figure 5 but only that portion exterior to the cone boundary; it does not include the white region above the truncation but inside the supercone. The outside test is $\mathbf{A}\cdot(\mathbf{C}-\mathbf{U}) \geq |\mathbf{C}-\mathbf{U}|\cos\theta$.

2. $\mathbf{C}$ is outside the sphere-swept volume when it is below the plane through $\mathbf{E}$ and perpendicular to the cone axis. The center is in the green region of Figure 5. The outside test is $\mathbf{A}\cdot(\mathbf{C}-\mathbf{E}) < 0$, which is equivalent to $\mathbf{A}\cdot(\mathbf{C}-\mathbf{V}) < -r$.

3. $\mathbf{C}$ is outside the sphere-swept volume when it is above the plane through $\overline{\mathbf{E}}$ and perpendicular to the cone axis. The center is in the white region of Figure 5 that is in the supercone. The outside test is $\mathbf{A} \cdot (\mathbf{C} - \overline{\mathbf{E}}) > 0$, which is equivalent to $\mathbf{A}\dot{(}\mathbf{C} - \mathbf{V}) > h_{\max} + r$.

4. $\mathbf{C}$ is inside the sphere-swept volume when it is between the planes through $\mathbf{M}$ and $\overline{\mathbf{M}}$ and perpendicular to the cone axis. The center is in the red or orange regions of Figure 5. The inside tests are $\mathbf{A} \cdot (\mathbf{C} - \mathbf{M}) \geq 0$ and $\mathbf{A} \cdot (\mathbf{C} - \overline{\mathbf{M}}) \leq 0$, which are equivalent to $-r \sin\theta \leq \mathbf{A} \cdot (\mathbf{C} - \mathbf{V}) \leq h_{\max} - r \sin\theta$.

5. $\mathbf{C}$ is inside the sphere-swept volume if it is above the $\mathbf{E}$-plane and below the $\mathbf{M}$-plane. The center is in the blue or yellow regions of Figure 5. The test for containment is $|\mathbf{\Delta}|^2 \leq r^2$.

6. $\mathbf{C}$ is inside the sphere-swept volume if it is above the $\overline{\mathbf{E}}$-plane, below the $\overline{\mathbf{M}}$-plane and a distance d from the cone axis with $d = |\mathbf{A} \times \overline{\mathbf{\Delta}}| \leq h_{\max} \tan\theta$. The center is in the violet region of Figure 5.

7. The final test is whether $\mathbf{C}$ is in a yellow or a blue region above the $\overline{\mathbf{M}}$-plane of Figure 5. This requires a simple distance comparison between the sphere radius and the distance from $\mathbf{C}$ to $\overline{\mathbf{K}}$. The test

$$|\mathbf{C} - \overline{\mathbf{K}}|^2 = (\mathbf{A} \cdot \overline{\mathbf{\Delta}})^2 + \left(|\mathbf{A} \times \overline{\mathbf{\Delta}}| - h_{\max} \tan\theta\right)^2 \leq r^2$$

is true if and only if $\mathbf{C}$ is inside the sphere centered at $\overline{\mathbf{K}}$, in the blue region. When the test is false, $\mathbf{C}$ is in the yellow region and outside the sphere-swept volume.

Listing 3 contains pseudocode for the test-intersection query.

Listing 3. Pseudocode for the test-intersection query between a sphere and a finite cone. The function returns `true` iff there is an intersection.

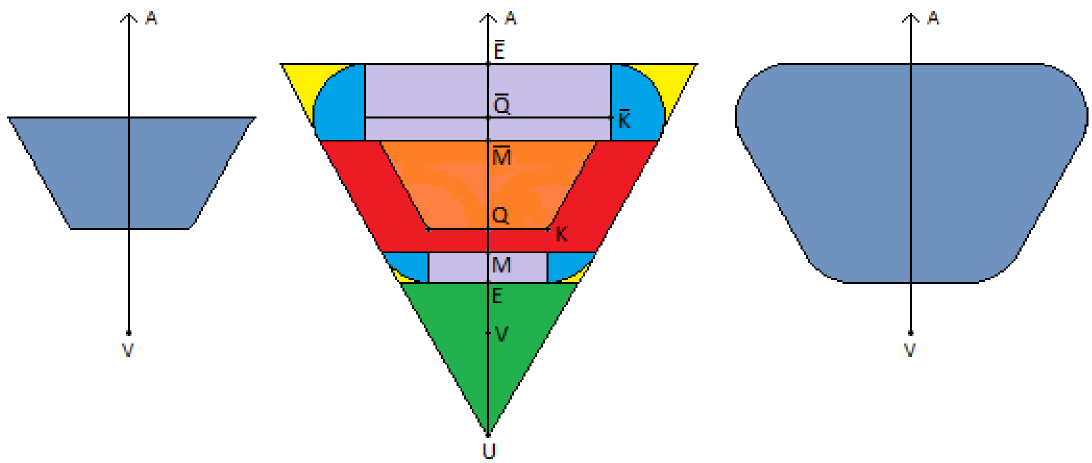
```
bool SphereIntersectsFiniteCone(Sphere sphere, Cone cone)
{
    Vector3 U = cone.vertex - (sphere.radius * cone.sinReciprocal) * cone.axis;
    Vector3 CmU = sphere.center - U;
    Real AdCmU = Dot(cone.axis, CmU);
    if (AdCmU > 0)
    {
        Real sqrLengthCmU = Dot(CmU, CmU);
        if (AdCmU * AdCmU >= sqrLengthCmU * cone.cosAngleSqr)
        {
            // The center is inside the supercone.
            Vector3 CmV = sphere.center - cone.vertex;
            Real AdCmV = Dot(cone.axis, CmV);
            if (AdCmV < -sphere.radius)
            {
                // The center is outside the sphere-swept volume (green region).
                return false;
            }

            if (AdCmV > cone.maxHeight + sphere.radius)
            {
                // The center is outside the sphere-swept volume (white region).
                return false;
            }

            Real rSinAngle = sphere.radius * cone.sinAngle;
            if (AdCmV >= -rSinAngle)
            {
                if (AdCmV <= cone.maxHeight - rSinAngle)
                {
                    // The center is inside the sphere-swept volume (red or orange region).
                    return true;
                }
```

```
            else
            {
                Vector3 barD = CmV - cone.maxHeight * cone.axis;  // = C - barQ = C - V - hmax * A
                Real lengthAxBarD = Length(Cross(cone.axis, barD));
                Real hmaxTanAngle = cone.maxHeight * cone.tanAngle;
                if (lengthAxBarD <= hmaxTanAngle)
                {
                    // The center is inside the sphere-swept volume (top violet region).
                    return true;
                }

                Real AdBarD = AdCmV - cone.maxHeight;  // = Dot(A, C - barQ) = Dot(A, C - V) - hmax
                Real diff = lengthAxBarD - hmaxTanAngle;
                Real sqrLengthCmBarK = AdBarD * AdBarD + diff * diff;
                if (sqrLengthCmBarK <= sphere.radiusSqr)
                {
                    // The center is inside the sphere-swept volume (top blue region).
                    return true;
                }
                else
                {
                    // The center is outside the sphere-swept volume (top yellow region).
                    return false;
                }
            }
        }
        else
        {
            Real sqrLengthCmV = Dot(CmV, CmV);
            if (sqrLengthCmV <= sphere.radiusSqr)
            {
                // The center is inside the sphere-swept volume (bottom blue region).
                return true;
            }
            else
            {
                // The center is outside the sphere-swept volume (bottom yellow region).
                return false;
            }
        }
    }
}

// The sphere center is outside the supercone (white region).
return false;
}
```

5 Intersection of a Sphere with a Cone Frustum

Figure 6 shows regions of interest in a cross section of the cone. The cross section lives in a plane containing the sphere center **C**, the cone vertex **V** and the cone axis direction **A**.

Figure 6. Left: The solid cone frustum. Right: The sphere-swept solid cone frustum. Middle: A partitioning of the sphere-swept solid cone frustum. The sphere-swept volume is formed by the orange, red, blue and violet regions. The yellow and green regions are outside the sphere-swept volume. The points are the same as those defined in Figures 3 and 5.



The conditions for when the sphere will or will not intersect the cone frustum are a combination of some of the infinite truncated cone tests and some of the finite cone tests. Listing 4 contains pseudocode for testing whether a sphere intersects a cone frustum.

Listing 4. Pseudocode for the test-intersection query between a sphere and a cone frustum. The function returns `true` iff there is an intersection.

```
bool SphereIntersectsConeFrustum(Sphere sphere, Cone cone)
{
    Vector3 U = cone.vertex - (sphere.radius * cone.sinReciprocal) * cone.axis;
    Vector3 CmU = sphere.center - U;
    Real AdCmU = Dot(cone.axis, CmU);
    if (AdCmU > 0)
    {
        Real sqrLengthCmU = Dot(CmU, CmU);
        if (AdCmU * AdCmU >= sqrLengthCmU * cone.cosAngleSqr)
        {
            // The center is inside the supercone.
            Vector3 CmV = sphere.center - cone.vertex;
            Real AdCmV = Dot(cone.axis, CmV);
            if (AdCmV < cone.minHeight - sphere.radius)
            {
                // The center is outside the sphere-swept volume (green region).
                return false;
            }

            if (AdCmV > cone.maxHeight + sphere.radius)
            {
                // The center is outside the sphere-swept volume (white region).
                return false;
            }

            Real rSinAngle = sphere.radius * cone.sinAngle;
            if (AdCmV >= cone.minHeight - rSinAngle)
            {
                if (AdCmV <= cone.maxHeight - rSinAngle)
```

```
                {
                    // The center is inside the sphere-swept volume (red or orange region).
                    return true;
                }
                else
                {
                    Vector3 barD = CmV - cone.maxHeight * cone.axis;  // = C - barQ = C - V - hmax * A
                    Real lengthAxBarD = Length(Cross(cone.axis, barD));
                    Real hmaxTanAngle = cone.maxHeight * cone.tanAngle;
                    if (lengthAxBarD <= hmaxTanAngle)
                    {
                        // The center is inside the sphere-swept volume (top violet region).
                        return true;
                    }

                    Real AdBarD = AdCmV - cone.maxHeight;  // = Dot(A, C - barQ) = Dot(A, C - V) - hmax
                    Real diff = lengthAxBarD - hmaxTanAngle;
                    Real sqrLengthCmBarK = AdBarD * AdBarD + diff * diff;
                    if (sqrLengthCmBarK <= sphere.radiusSqr)
                    {
                        // The center is inside the sphere-swept volume (top blue region).
                        return true;
                    }
                    else
                    {
                        // The center is outside the sphere-swept volume (top yellow region).
                        return false;
                    }
                }
            }
            else
            {
                Vector3 D = CmV - cone.minHeight * cone.axis;  // = C - Q = C - V - hmin * A
                Real lengthAxD = Length(Cross(cone.axis, D));
                Real hminTanAngle = cone.minHeight * cone.tanAngle;
                if (lengthAxD <= hminTanAngle)
                {
                    // The center is inside the sphere-swept volume (violet region).
                    return true;
                }

                Real AdD = AdCmV - cone.minHeight;  // = Dot(A, C - Q) = Dot(A, C - V) - hmin
                Real diff = lengthAxD - hminTanAngle;
                Real sqrLengthCmK = AdD * AdD + diff * diff;
                if (sqrLengthCmK <= sphere.radiusSqr)
                {
                    // The center is inside the sphere-swept volume (blue region).
                    return true;
                }
                else
                {
                    // The center is outside the sphere-swept volume (yellow region).
                    return false;
                }
            }
        }
    }

    // The center is outside the supercone (white region).
    return false;
}
```

6 Implementation

An implementation of the algorithm in GTE is IntrSphere3Cone3.h. There is also a sample application whose path is GeometricTools/GTE/Samples/Intersection/IntersectSphereCone.