Distance Between Point and Line, Ray, or Line Segment

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1 Discussion

The following construction applies in any dimension, not just in 3D. Let the test point be \( P \). A line is parameterized as \( L(t) = B + tM \) where \( B \) is a point on the line, \( M \) is the line direction, and \( t \in \mathbb{R} \). A ray is of the same form but with restriction \( t \geq 0 \). A line segment is restricted even further with \( t \in [0, 1] \). The end points of the line segment are \( B \) and \( B + M \).

The closest point on the line to \( P \) is the projection of \( P \) onto the line, \( Q = B + t_0M \), where

\[
t_0 = \frac{M \cdot (P - B)}{M \cdot M}.
\]

The distance from \( P \) to the line is

\[
D = |P - (B + t_0M)|.
\]

If \( t_0 \leq 0 \), then the closest point on the ray to \( P \) is \( B \). For \( t_0 > 0 \), the projection \( B + t_0M \) is the closest point. The distance from \( P \) to the ray is

\[
D = \begin{cases} 
|P - B|, & t_0 \leq 0 \\
|P - (B + t_0M)|, & t_0 > 0 
\end{cases}
\]

Finally, if \( t_0 > 1 \), then the closest point on the line segment to \( P \) is \( B + M \). The distance from \( P \) to the line segment is

\[
D = \begin{cases} 
|P - B|, & t_0 \leq 0 \\
|P - (B + t_0M)|, & 0 < t_0 < 1 \\
|P - (B + M)|, & t_0 \geq 1 
\end{cases}
\]

The division by \( M \cdot M \) is the most expensive algebraic operation. The implementation should defer the division as late as possible.