Computing Impulse Forces for Colliding Contact

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Contents

This is a note about the derivation described in *Game Physics, 2nd edition* [1, Section 6.2.2] about computing impulsive forces for collision response at colliding contacts.

1 Impulses as Described in the Game Physics Book

Consider two rigid bodies named A and B that have colliding contact at the he point $\mathcal{P}_0$ at simulation time t_0. The normal vector at the contact point on body B is $\boldsymbol{N}_0$. The contact point on body A before the collision is located at $\mathcal{P}_A(t)$ for $t \leq t_0$; it is the case that $\mathcal{P}_A(t_0) = \mathcal{P}_0$. Similarly, the contact point on body B before the collision is located at $\mathcal{P}_B(t)$ for $t \leq t_0$; it is the case that $\mathcal{P}_B(t_0) = \mathcal{P}_0$. The normal vector at $\mathcal{P}_B(t)$ is $\boldsymbol{N}(t)$ for $t \leq t_0$ with $\boldsymbol{N}(t_0) = \boldsymbol{N}_0$.

Before the contact, the signed distance between the points measured in the normal direction is

$$d(t) = \boldsymbol{N}(t) \cdot (\mathcal{P}_A(t) - \mathcal{P}_B(t)) \tag{1}$$

Define the velocities $\mathcal{V}_A(t) = \dot{\mathcal{P}}_A(t)$ and $\mathcal{V}_B(t) = \dot{\mathcal{P}}_B(t)$, where the dot indicates a derivative with respect to time t. Using the chain rule for differentiation, the velocity component in the normal direction is

$$\dot{d}(t) = \boldsymbol{N}(t) \cdot (\mathcal{V}_A(t) - \mathcal{V}_B(t)) + \dot{\boldsymbol{N}}(t) \cdot (\mathcal{P}_A(t) - \mathcal{P}_B(t)) \tag{2}$$

At the instant of contact, $d(t_0) = 0$ and $\dot{d}(t_0) = \boldsymbol{N}_0 \cdot (\mathcal{V}_A(t_0) - \mathcal{V}_B(t_0)$. To be a point of colliding contact, it is necessary that $\dot{d}(t_0) < 0$.

The velocities at $\mathcal{P}_A(t)$ and $\mathcal{P}_B(t)$ are

$$\mathcal{V}_A(t) = \boldsymbol{v}_A(t) + \boldsymbol{w}_A(t) \times \boldsymbol{r}_A(t),\ \mathcal{V}_B(t) = \boldsymbol{v}_B(t) + \boldsymbol{w}_B(t) \times \boldsymbol{r}_B(t) \tag{3}$$

where $\boldsymbol{r}_A(t) = \mathcal{P}_A(t) - \mathcal{X}_A(t)$ and $\boldsymbol{r}_B(t) = \mathcal{P}_B(t) - \mathcal{X}_B(t)$ with $\mathcal{X}_A(t)$ and $\mathcal{X}_B$ the positions of the centers of mass of the bodies. The linear velocities of the centers of mass are $\boldsymbol{v}_A(t)$ and $\boldsymbol{v}_B(t)$ and the angular velocities of the centers of mass are $\boldsymbol{w}_A(t)$ and $\boldsymbol{w}_B(t)$. The velocity in the normal direction at contact is therefore

$$\dot{d}(t_0) = \boldsymbol{N}_0 \cdot ((\boldsymbol{v}_A(t_0) + \boldsymbol{w}_A(t_0) \times \boldsymbol{r}_A(t_0)) - (\boldsymbol{v}_B(t_0) + \boldsymbol{w}_B(t_0) \times \boldsymbol{r}_B(t_0))) \tag{4}$$

where $\boldsymbol{r}_A(t_0) = \mathcal{P}_0 - \mathcal{X}_A(t_0)$ and $\boldsymbol{r}_B(t_0) = \mathcal{P}_0 - \mathcal{X}_B(t_0)$.

We need to prevent interpenetration of the two bodies. This can be accomplished by *impulses*, which is an instantaneous change in the velocities $\mathcal{V}_A(t)$ and $\mathcal{V}_B(t)$ at t_0 as t varies from values smaller than t_0 to values larger than t_0. The positions are continous functions at t_0 but the velocities have a discontinuity at t_0. To simplify the reading, I will omit the explicit dependence on t_0 from the equations.

Using the notation of one-sided limits, at t_0 the *preimpulse velocities* at $\mathcal{P}_0$ are $\mathcal{V}_A^-$ and $\mathcal{V}_B^-$ and the *postimpulse velocities* at $\mathcal{P}_0$ are $\mathcal{V}_A^+$ and $\mathcal{V}_B^+$. The corresponding linear and angular velocities also have preimpulse and postimpulse counterparts. Using the same notation forone-sided limits,

$$\mathcal{V}_A^- = \boldsymbol{v}_A^- + \boldsymbol{w}_A^- \times \boldsymbol{r}_A,\ \mathcal{V}_A^+ = \boldsymbol{v}_A^+ + \boldsymbol{w}_A^+ \times \boldsymbol{r}_A \tag{5}$$

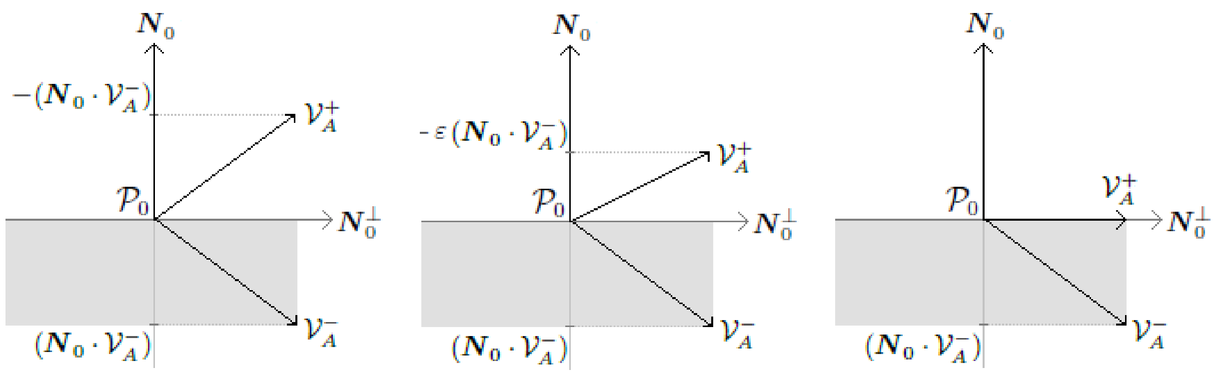
and

$$\mathcal{V}_B^- = \boldsymbol{v}_B^- + \boldsymbol{w}_B^- \times \boldsymbol{r}_B,\ \mathcal{V}_B^+ = \boldsymbol{v}_B^+ + \boldsymbol{w}_B^+ \times \boldsymbol{r}_B \tag{6}$$

The vectors $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$ do not depend on velocities. They are continuous at t_0 and do not require one-sided-limit notation.

The preimpulse velocity relative to $\mathcal{P}_0$ is $\mathcal{V}^- = \mathcal{V}_A^- - \mathcal{V}_B^-$ and the postimpulse velocity relative to $\mathcal{P}_0$ is $\mathcal{V}^+ = \mathcal{V}_A^- - \mathcal{V}_B^-$. One choice to prevent interpenetration of the two bodies is to reflect the preimpulse about the normal $\boldsymbol{N}_0$, possibly including a *coefficient of restitution* $\varepsilon \in [0, 1]$ that controls loss of kinetic energy when the contact occurs. Figure 1 illustrates this.

Figure 1. Reflections of the preimpulse velocity through the normal $\boldsymbol{N}_0$. In (a), there is no loss of kinetic energy ($\varepsilon = 1$). In (b), there is some loss of kinetic energy ($0 < \varepsilon < 1$). In (c), there is total loss of kinetic energy.



(a) (b) (c)

The preimpulse velocity is decomposed into

$$\mathcal{V}_A^- = \boldsymbol{N}_0^\perp + (\boldsymbol{N}_0 \cdot \mathcal{V}_A^-)\boldsymbol{N}_0 \tag{7}$$

where the right-most term is the component in the $\boldsymbol{N}_0$ direction. The vector $\boldsymbol{N}_0^\perp$ is the remainder after removing the normal component. The postimpulse velocity is the reflection of the preimpulse velocity through the line $\mathcal{P}_0 + s\boldsymbol{N}_0^\perp$,

$$\mathcal{V}_A^+ = \boldsymbol{N}_0^\perp - \varepsilon(\boldsymbol{N}_0 \cdot \mathcal{V}_A^-)\boldsymbol{N}_0 \tag{8}$$

The difference is

$$\mathcal{V}_A^+ - \mathcal{V}_A^- = -(1 + \varepsilon)(\boldsymbol{N}_0 \cdot \mathcal{V}_A^-)\boldsymbol{N}_0 \tag{9}$$

To obtain a reflection, apply an impulsive force $\boldsymbol{F} = f\boldsymbol{N}_0$ that affects body A at $\mathcal{P}_0$ for some scalar f to be determined. The impulsive force has a contribution that changes the linear velocity of the center of mass, $f\boldsymbol{N}_0/m_A$, where m_A is the mass of body A. The preimpulse and postimpulse linear velocities are related by

$$\boldsymbol{v}_A^+ = \boldsymbol{v}_A^- + \frac{f\boldsymbol{N}_0}{m_A} \tag{10}$$

The corresponding change in linear momentum is $\boldsymbol{p}_A^+ = \boldsymbol{p}_A^- + f\boldsymbol{N}_0$. The impulsive force also has a contribution that changes the angular velocity of the body via an impulsive torque, $J_A^{-1}(\boldsymbol{r}_A \times f\boldsymbol{N}_0)$, where J_A is the world inertia tensor at the contact time. The preimpulse and postimpulse angular velocities are related by

$$\boldsymbol{w}_A^+ = \boldsymbol{w}_A^- + J_A^{-1}(\boldsymbol{r}_A \times f\boldsymbol{N}_0) \tag{11}$$

The corresponding change in angular momentum is $\boldsymbol{L}_A^+ = \boldsymbol{L}_A^- + \boldsymbol{r}_A \times f\boldsymbol{N}_0$. Substituting equations (10) and (11) into the velocity equations (5) leads to

$$\mathcal{V}_A^+ = \mathcal{V}_A^- + f\left(\frac{\boldsymbol{N}_0}{m_A} + \left(J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{N}_0)\right) \times \boldsymbol{r}_A\right) \tag{12}$$

A similar construction can be used for body B. The analogy for equation (9) is

$$\mathcal{V}_B^+ - \mathcal{V}_B^- = -(1+\varepsilon)(\boldsymbol{N}_0 \cdot \mathcal{V}_B^-)\boldsymbol{N}_0 \tag{13}$$

The opposite-direction impulsive force $-\boldsymbol{F}$ is applied to body B. The construction produces $\boldsymbol{v}_B^+ = \boldsymbol{v}_B^- - f\boldsymbol{N}_0/m_B$, $\boldsymbol{p}_B^+ = \boldsymbol{p}_B^- - f\boldsymbol{N}_0$, $\boldsymbol{w}_B^+ = \boldsymbol{w}_B^- - J_B^{-1}(\boldsymbol{r}_B \times f\boldsymbol{N}_0)$, $\boldsymbol{L}_B^+ = \boldsymbol{L}_B^- - \boldsymbol{r}_B \times f\boldsymbol{N}_0$ and

$$\mathcal{V}_B^+ = \mathcal{V}_B^- - f\left(\frac{\boldsymbol{N}_0}{m_B} + \left(J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{N}_0)\right) \times \boldsymbol{r}_B\right) \tag{14}$$

Equations (12) and (14) are combined to obtain the relationship between preimpulse relative velocity and postimpulse relative velocity,

$$\mathcal{V}^+ = \mathcal{V}^- + f\left(\frac{\boldsymbol{N}_0}{m_A} + \frac{\boldsymbol{N}_0}{m_B} + (J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{N}_0)) \times \boldsymbol{r}_A + (J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{N}_0)) \times \boldsymbol{r}_B\right) \tag{15}$$

Compute the dot product of equation (15) with $\boldsymbol{N}_0$ to obtain

$$\boldsymbol{N}_0 \cdot \mathcal{V}^+ = \boldsymbol{N}_0 \cdot \mathcal{V}^- + f\left(m_A^{-1} + m_B^{-1} + (\boldsymbol{r}_A \times \boldsymbol{N}_0))^{\mathsf{T}} J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{N}_0) + (\boldsymbol{r}_B \times \boldsymbol{N}_0))^{\mathsf{T}} J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{N}_0)\right) \tag{16}$$

Subtracting equations (9) and (13), we obtain

$$\boldsymbol{N}_0 \cdot \mathcal{V}^+ = -\varepsilon \boldsymbol{N}_0 \cdot \mathcal{V}^- \tag{17}$$

Finally, substitute equations (17), (5) and (6) into equation (16) to obtain the magnitude f,

$$f = \frac{-(1+\varepsilon)(\boldsymbol{N}_0 \cdot (\boldsymbol{v}_A^- - \boldsymbol{v}_B^-) + (\boldsymbol{w}_A^- \cdot (\boldsymbol{r}_A \times \boldsymbol{N}_0) - \boldsymbol{w}_B^- \cdot (\boldsymbol{r}_B \times \boldsymbol{N}_0)))}{m_A^{-1} + m_B^{-1} + (\boldsymbol{r}_A \times \boldsymbol{N}_0)^{\mathsf{T}} J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{N}_0) + (\boldsymbol{r}_B \times \boldsymbol{N}_0)^{\mathsf{T}} J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{N}_0)} \tag{18}$$

Once f is computed from preimpulse quantities, the postimpulse quantites can be computed and the numerical differential equation solver for the equations of motion can procede with the new state.

2 Impulses Using a Variation of the Algorithm

The algorithm of the previous section appears to work well for convex polyhedra. The Wild Magic physics sample BouncingSpheres is a simulation whose rigid bodies are solid spheres contained inside a room bounded by immovable walls. The colliding contacts of the spheres occur in a manner that looks realistic, but none of the spheres have angular velocity. I had ported the sample to use the Geometric Tools code base and now decided that the simulation would look more realistic if the spheres have angular velocity and angular momentum. I also decided that adding friction forces when the spheres are rolling and spinning on the ground plane would help with the realism.

Adding angular momentum to the spheres did not lead to spinning spheres. I tracked the problem to the computation of postimpulse angular momentum. After computing the impulse magnitudes of equation (18), the postimpulse angular momenta were the same as the preimpulse angular momenta. Consequently, the angular velocities remained constant. Still I expected the spheres to spin, but they did not.

The physical simulation uses the theoretical spheres, not convex polyhedral approximations. The first problem is that for colliding contact of two spheres, the centers of mass $\mathcal{X}_A$ and $\mathcal{X}_B$ and the contact point $\mathcal{P}_0$ are collinear, the common line having direction $\boldsymbol{N}_0$. The angular momentum updates are therefore

$$\begin{aligned}\boldsymbol{L}_A^+ &= \boldsymbol{L}_A^- + \boldsymbol{r}_A \times (f\boldsymbol{N}_0) = \boldsymbol{L}_A^- + f(\mathcal{P}_0 - \mathcal{X}_A) \times \boldsymbol{N}_0 = \boldsymbol{L}_A^- \\ \boldsymbol{L}_B^+ &= \boldsymbol{L}_B^- - \boldsymbol{r}_B \times (f\boldsymbol{N}_0) = \boldsymbol{L}_B^- + f(\mathcal{P}_0 - \mathcal{X}_B) \times \boldsymbol{N}_0 = \boldsymbol{L}_B^-\end{aligned} \tag{19}$$

The vectors $\mathcal{P}_0 - \mathcal{X}_A$ and $\mathcal{P}_0 - \mathcal{X}_B$ are parallel to $\boldsymbol{N}_0$, so the cross product terms are zero. The conclusion is that for colliding spheres, the angular momentum is constant throughout time and does not change even with the impulses. The sample application has an external torque function that returns the zero vector, so during the time between two consecutive colliding contacts, the equation of motion $d\boldsymbol{L}/dt = \boldsymbol{\tau} = \boldsymbol{0}$ integrates to $\boldsymbol{L}(t) = \boldsymbol{L}_0$, which means angular momentum is constant. One might have thought the impulses would lead to a piecewise constant angular momentum function over time, but as shown previously, the impulses do not change the value.

The relationship between angular velocity $\boldsymbol{w}(t)$ and angular momentum $\boldsymbol{L}(t)$ is $\boldsymbol{L}(t) = J(t)\boldsymbol{w}(t)$, where $J(t)$ is the world inertia tensor for the rigid body. The world inertia tensor is $J(t) = R(t)J_{\text{body}}R(t)^{\mathsf{T}}$ where J_{body} is the body inertia tensor and $R(t)$ is the orientation matrix of the rigid body. For a sphere, J_{body} is kI where I is the identity matrix and where k is a positive constant that depends on mass density and radius. This implies $J(t) = R(t)kIR(t)^{\mathsf{T}} = kI$; that is, the world inertia tensor is constant over time. The angular velocity is $\boldsymbol{w}(t) = \boldsymbol{L}(t)/k$, which implies for a sphere that $\boldsymbol{w}(t)$ is constant over time.

To use the impulse-based approach and to eliminate the problem with the zero-valued cross product terms, the impulse forces need to be chosen with a direction that is not necessarily the normal direction $\boldsymbol{N}_0$. The choice of force $f\boldsymbol{N}_0$ was motivated by wanting to reflect $\mathcal{V}_A^-$ to $\mathcal{V}_A^+$ through a plane perpendicular to $\boldsymbol{N}_0$; see figure 1(a). The construction was designed to obtain a postimpulse velocity whose component in the normal direction is the negative of the preimpulse velocity normal component. However, matching this does not guarantee that $\mathcal{V}_A^+$ is in the plane spanned by $\boldsymbol{N}_0$ and $\mathcal{V}_A^-$, so in a sense figure 1 is misleading because it was drawn in 2D!

Equation (17) states that $\boldsymbol{N}_0 \cdot \mathcal{V}^+ = -\varepsilon \boldsymbol{N}_0 \cdot \mathcal{V}^-$. This constraint remains, but to ensure that the reflected postimpulse velocity is in the plane spanned by $\boldsymbol{N}_0$ and $\mathcal{V}^-$, the projections of the preimpulse and postimpulse velocities onto the plane perpendicular to $\boldsymbol{N}_0$ must have the same direction. If $\boldsymbol{T}_0$ is a unit-length vector in the reflection plane having the same direction as $\boldsymbol{V}^-$, then I added a new constraint

$$\boldsymbol{T}_0 \cdot \mathcal{V}^+ = \boldsymbol{T}_0 \cdot \mathcal{V}^- \tag{20}$$

Choosing $\boldsymbol{T}_1 = \boldsymbol{N}_0 \times \boldsymbol{T}_0$, it is the case that $\boldsymbol{T}_1 \cdot \mathcal{V}^- = 0$. The postimpulse velocity must also be perpendicular to $\boldsymbol{T}_1$. The constraints are

$$\boldsymbol{T}_1 \cdot \mathcal{V}^+ = \boldsymbol{T}_1 \cdot \mathcal{V}^- = 0 \tag{21}$$

The set of vectors $\{\boldsymbol{N}_0, \boldsymbol{T}_0, \boldsymbol{T}_1\}$ is a right-handed orthonormal basis.

The impulse force is now chosen to be

$$\boldsymbol{F} = \phi \boldsymbol{N}_0 + \psi \boldsymbol{T}_0 + \xi \boldsymbol{T}_1 \tag{22}$$

The relationships between the preimpulse and postimpulse quantities are

$$\begin{array}{rclcrcl} \boldsymbol{v}_A^+ &=& \boldsymbol{v}_A^- + \boldsymbol{F}/m_A & \qquad & \boldsymbol{v}_B^+ &=& \boldsymbol{v}_B^- - \boldsymbol{F}/m_B \\ \boldsymbol{w}_A^+ &=& \boldsymbol{w}_A^- + J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{F}) & & \boldsymbol{w}_B^+ &=& \boldsymbol{w}_B^- - J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{F}) \\ \boldsymbol{p}_A^+ &=& \boldsymbol{p}_A^- + \boldsymbol{F} & & \boldsymbol{p}_B^+ &=& \boldsymbol{p}_B^- - \boldsymbol{F} \\ \boldsymbol{L}_A^+ &=& \boldsymbol{L}_A^- + \boldsymbol{r}_A \times \boldsymbol{F} & & \boldsymbol{L}_B^+ &=& \boldsymbol{L}_B^- - \boldsymbol{r}_B \times \boldsymbol{F} \end{array} \tag{23}$$

The equivalent of equation (15) is

$$\mathcal{V}^+ = \mathcal{V}^- + \left(\frac{\boldsymbol{F}}{m_A} + \frac{\boldsymbol{F}}{m_B} + (J_A^{-1}(\boldsymbol{r}_A \times \boldsymbol{F})) \times \boldsymbol{r}_A + (J_B^{-1}(\boldsymbol{r}_B \times \boldsymbol{F})) \times \boldsymbol{r}_B \right) \tag{24}$$

Define $\lambda = m_A^{-1} + n_B^{-1}$. Dotting equation (24) with the 3 basis vectors leads to a linear system of 3 equations in 3 unknowns,

$$\begin{bmatrix} \lambda + S(\boldsymbol{N}_0, \boldsymbol{N}_0) & S(\boldsymbol{N}_0, \boldsymbol{T}_0) & S(\boldsymbol{N}_0, \boldsymbol{T}_1) \\ S(\boldsymbol{T}_0, \boldsymbol{N}_0) & \lambda + S(\boldsymbol{T}_0, \boldsymbol{T}_0) & S(\boldsymbol{T}_0, \boldsymbol{T}_1) \\ S(\boldsymbol{T}_1, \boldsymbol{N}_0) & S(\boldsymbol{T}_1, \boldsymbol{T}_0) & \lambda + S(\boldsymbol{T}_0, \boldsymbol{T}_0) \end{bmatrix} \begin{bmatrix} \phi \\ \psi \\ \xi \end{bmatrix} = \begin{bmatrix} -(1+\varepsilon)\mathcal{V}^- \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

where

$$S(\boldsymbol{X}, \boldsymbol{Y}) = (\boldsymbol{r}_A \times \boldsymbol{X})^\mathsf{T} J_A^{-1} (\boldsymbol{r}_A \times \boldsymbol{Y}) + (\boldsymbol{r}_B \times \boldsymbol{X})^\mathsf{T} J_B^{-1} (\boldsymbol{r}_B \times \boldsymbol{Y}) \tag{26}$$

Let C be the 3×3 matrix of the linear system; then

$$C = \lambda I + \begin{bmatrix} S(\boldsymbol{N}_0, \boldsymbol{N}_0) & S(\boldsymbol{N}_0, \boldsymbol{T}_0) & S(\boldsymbol{N}_0, \boldsymbol{T}_1) \\ S(\boldsymbol{T}_0, \boldsymbol{N}_0) & S(\boldsymbol{T}_0, \boldsymbol{T}_0) & S(\boldsymbol{T}_0, \boldsymbol{T}_1) \\ S(\boldsymbol{T}_1, \boldsymbol{N}_0) & S(\boldsymbol{T}_1, \boldsymbol{T}_0) & S(\boldsymbol{T}_0, \boldsymbol{T}_0) \end{bmatrix} = \lambda I + Q_A^\mathsf{T} J_A^{-1} Q_A + Q_B^\mathsf{T} J_B^{-1} Q_B \tag{27}$$

where I is the 3×3 identity and Q_A and Q_B are 3×3 matrices shown next with their columns listed,

$$Q_A = \begin{bmatrix} \boldsymbol{r}_A \times \boldsymbol{N}_0 & \boldsymbol{r}_A \times \boldsymbol{T}_0 & \boldsymbol{r}_A \times \boldsymbol{T}_1 \end{bmatrix}, \quad Q_B = \begin{bmatrix} \boldsymbol{r}_B \times \boldsymbol{N}_0 & \boldsymbol{r}_B \times \boldsymbol{T}_0 & \boldsymbol{r}_B \times \boldsymbol{T}_1 \end{bmatrix} \tag{28}$$

Representing $\boldsymbol{r}_A = k_0 \boldsymbol{N}_0 + k_1 \boldsymbol{T}_0 + k_2 \boldsymbol{T}_1$, Q_A becomes

$$Q_A = \begin{bmatrix} 0 & -k_2 & k_1 \\ k_2 & 0 & -k_0 \\ -k_1 & k_0 & 0 \end{bmatrix} \tag{29}$$

which is a skew-symmetric matrix with real-valued eigenvalue 0 and corresponding eigenvector (k_0, k_1, k_2). The other eigenvalues are complex valued. For any nonzero vector $\boldsymbol{y}$ it must be that $\boldsymbol{y}^\mathsf{T} Q_A \boldsymbol{y} \geq 0$ which means Q_A is positive semidefinite. A similar argument shows that Q_B is positive semidefinite. The matrices J_A^{-1} and J_B^{-1} are positive definite. Therefore, the matrices $Q_A^\mathsf{T} J_A^{-1} Q_A$ and $Q_B^\mathsf{T} J_B^{-1} Q_B$ are positive semidefinite. The matrix λI is positive definite. Combining these results, the matrix C is positive definite which ensures that the matrix system always has a solution.

The angular momentum update is

$$\boldsymbol{L}_A^+ = \boldsymbol{L}_A^- + \boldsymbol{r}_A \times (\phi \boldsymbol{N}_0 + \psi \boldsymbol{T}_0 + \xi \boldsymbol{T}_1) \tag{30}$$

When ψ or ξ are not zero, the postimpulse angular momentum does in fact change, leading to a piecewise constant function whose discontinuities occur at times of colliding contact. This means that the angular momenta can change when two spheres collide. The BouncingSpheres sample application uses this variation.

References

[1] David Eberly. *Game Physics.* CRC Press, Taylor & Francis Group LLC, Boca Raton, FL, 2nd edition, 2010.